Discussion

Comments on "Crack-size dependence of fracture toughness in transformation-toughened ceramics"

In a recent article Ikuma and Virkar [1] considered the use of indentation cracking in several transformation-toughened ceramics. In particular, they noted that for these materials are apparent value of fracture toughness (K_{IC}), as calculated using the approach of Evans and Charles [2], was dependent on indentation load. Ikuma and Virkar [1] interpreted these results on the presence of a residual compressive stress on the surface of the transformation-toughened materials. The variation of K_{IC} with indentation load has also been noted by this author but as will be shown, this effect was found not to be a result of surface residual stresses but was a limitation in the use of the Evans and Charles approach [2].

In previous work, there have been two approaches suggested [2, 3] for measuring K_{IC} from the size of the cracks that emanate from Vickers hardness indentations. Indeed, these two approaches have been compared and discussed [4–6]. It is worthwhile here, however, to show a comparison of the K_{IC} values calculated from these two approaches for a transformation-toughened Al_2O_3/ZrO_2 ceramic. The sintered specimen contained 30 vol % ZrO_2 (2.5 mol % Y_2O_3) and remained from a previous study [7]. In this material the ZrO_2 was completely retained in the tetragonal phase. Moreover, the specimen was annealed at 1400° C (16 h) and polished using 3 μm diamond paste. The comparison of the K_{IC} values is shown in Fig. 1. In a similar way to the data of Ikuma and Virkar [1], the values of K_{IC} calculated by the Evans and Charles technique [2, 8] increase when plotted against the square root of the indent crack size ($c^{1/2}$). In contrast, the approach of Lawn *et al.* [3] as calibrated by Anstis *et al.* [4] gives the fracture toughness as $K_{IC} = \chi P c^{-3/2}$, where $\chi = 0.016\ (E/H)^{1/2}$, P is the indentation load, E is Young's modulus and H is the hardness. As shown in Fig. 1, the K_{IC} date calculated by this technique are relatively independent of crack length and hence indentation load.

Ikuma and Virkar [1] showed that the use of the Evans and Charles approach [2] does not seem to present the same difficulty when applied to soda-lime silica glass. In referring back to the work of Evans and Charles [2], it is found that their universal plot of indentation data was non-linear, especially when the value of the ratio of the crack size to indentation size (c/a) was low. As pointed out later by Marshall and Evans [6], such a non-linearity does not occur for individual materials but rather the data follow curves of gradient $-3/2$ as expected by the analysis of Lawn *et al.* [3]. This implies that K_{IC} calculated by the latter technique [3] does not depend on c/a in the way suggested by Evans and Charles [2], but that the value of χ may vary from one material to another. The value of χ has been calibrated by Anstis *et al.* [4] for some materials, but for other materials it may be necessary to calibrate the indentation test if more accurate data is required. For example, in Fig. 1 it is noteworthy that there is also a large discrepancy in the magnitude of K_{IC} as calculated by the two approaches. From this discussion it appears that the approach of Evans and Charles [2] must be used with great caution, especially for brittle materials with the higher K_{IC} values. In terms of the data in Fig. 1, it is known that polishing and/or annealing substantially removes the residual surface stress for this specimen [7]. Therefore, in order to calculate a K_{IC} value which is independent of crack length, as expected for such a stress-free surface, the approach of Lawn and co-workers [3, 4] must be used. It is not necessary to consider the residual surface stress associated with the surface grinding process. It is, however, worth discussing what effect residual surface compression would have on indentation crack data.

Ikuma and Virkar [1] used an analysis of Marshall and Lawn [9] which shows that when an indentation crack is placed in a compressively stressed surface, the parameter $Pc^{-3/2}$ will increase linearly when plotted against $c^{1/2}$. As

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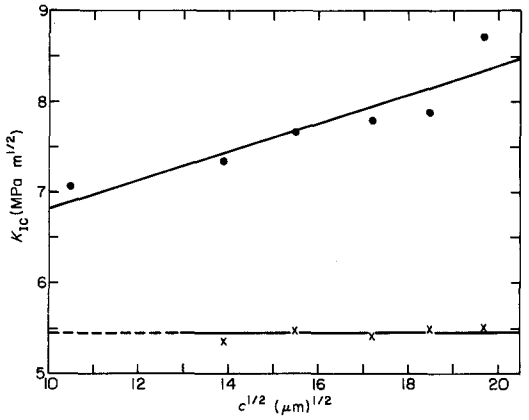

Figure 1 Comparison of K_{IC} values for a transformation-toughened Al_2O_3/30 vol % ZrO_2 specimen (annealed at 1400° C for 16 h), as calculated by the approach of Evans and Charles [2, 8] and that by Lawn and co-workers [3, 4]. ● after [2, 8]; x after [3, 4].

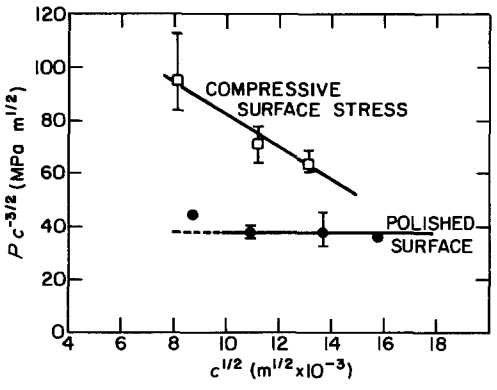

Figure 3 Comparison of $Pc^{-3/2}$ for a transformation-toughened β''-Al_2O_3/15 vol % ZrO_2 specimen with and without residual compressive surface stresses. The stresses were introduced using technique described in [10].

pointed out earlier for a stress-free surface, $P c^{-3/2}$ would be independent of crack length and proportional to K_{IC}. This analysis [9] is the origin of the idea of using plots such as Fig. 1 to study residual surface stresses. The analysis, however, assumed the compressive surface stress to be uniform over greater depths than the crack size. This is not expected for ground surfaces of transformation-toughened Al_2O_3/ZrO_2, as it has been shown that the residual stress gradient is steep and generally extends over small distances. For example, although stresses as high as 1 GPa have been measured on ground, transformation-toughened Al_2O_3/ZrO_2 surfaces, these stresses were found to extend only to depths of 10 to 25 μm [7]. Moreover, as indicated earlier these stresses could be removed by either polishing or annealing [7].

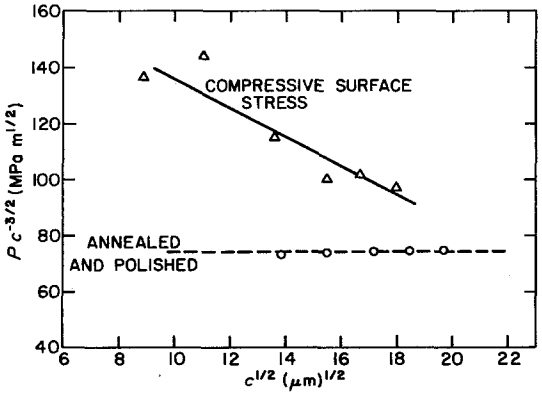

Figure 2 Comparison of $Pc^{-3/2}$ for a transformation-toughened Al_2O_3/30 vol % ZrO_2 specimen with and without residual compressive surface stresses. The stresses were introduced using technique described in [10].

Figs. 2 and 3 show the variation of $P c^{-3/2}$ for two transformation-toughened ceramics, i.e. Al_2O_3/30 vol % ZrO_2 and β''-Al_2O_3/15 vol % ZrO_2 specimens, in which surface compression was introduced by a special heat treatment [10]. In both cases, the data for a polished or annealed specimen are included for comparison. It is found that the value of $P c^{-3/2}$ is constant for the annealed or polished specimen, whereas it decreases with increasing crack length for the residually stressed specimens. The presence of the residual stresses was confirmed by an X-ray diffraction technique for the Al_2O_3/ZrO_2 specimens [10]. The decrease of $P c^{-3/2}$ can be understood when it is realized that in these materials the crack size are much greater than the depth of the compressive zone ($\sim$ 40 μm) [11], such that at the larger indentation loads the value of $P c^{-3/2}$ should approach that of a stress-free surface. In contrast to the work of Ikuma and Virkar [1], the apparent value of K_{IC} for a residually stressed surface, but calculated by the technique of Lawn and co-workers [3, 4] decreases with crack length and indentation load for transformation-toughened ceramics with residually stressed surfaces. There are other complications yet to be resolved in this type of work. For example, it is expected that residual stresses may also influence crack shape at indentations, and this type of problem awaits further analysis.

In conclusion, therefore, this discussion has

attempted to show the limitation of the Evans and Charles approach for transformation-toughened ceramics, especially when studying residually stressed surfaces. Indeed, the use of their approach can substantially alter the form of the indentation data, and hence confuse its interpretation.

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Reply to 'Comments on "Crack-size dependence of fracture toughness in transformation-toughened ceramics"'

In recent years, several investigators have used an indentation technique for determining fracture toughness, K_c, of brittle materials. Two approaches, both based upon the indenter crack approximated as a penny-shaped crack under point loading at the centre, have been suggested. Evans and Charles [1] used data from various materials of known fracture toughness (K_c) values and generated a master curve by plotting $(K_c\Phi/Ha^{1/2})(H/\Phi E)^{0.4}$ against c/a, where H is the hardness, E is the Young's modulus of elasticity, c is the crack radius and a is half the indent diagonal. The other approach, proposed by Anstis *et al.* [2] sets $K_c = \chi P/c^{3/2}$ with χ being a material dependent parameter. Green [3], in the preceding discussion of a recent paper by Ikuma and Virkar [4], suggests that the technique of Evans and Charles [1] leads to erroneous results and that one must use the method given by Anstis *et al.* [2]. The present authors have also noted that the two indentation techniques often give differing results and a discussion of this point is warranted. As will be shown in the following, however, the contention by Green [3] that the problem lies in the validity of the Evans and Charles [1] approach is without basis. The objective of our response is twofold. Firstly, we will examine Green's [3] data and assess the validity of the concept of the crack growing out of the zone of compression. Secondly, we will examine our data using both of the techniques and attempt to sort out the source of the discrepancy.

Green [3] suggests that the increase in K_c (as determined using the method of Evans and Charles [1]) with increasing $c^{1/2}$ observed in our and his work is the result of nonlinearity in the $(K_c\Phi/Ha^{1/2})(H/\Phi E)^{0.4}$ against c/a curve at low values of c/a. By contrast, the method of Anstis *et al.* [2] yielded K_c independent of c [3]. In Fig. 1, the data of Ikuma and Virkar [4] on ZrO_2 + 4.5 mol % Y_2O_3 is replotted. K_c calculated using the method of Anstis *et al.* [2] is also plotted in the same figure. Note that K_c against $c^{1/2}$ using this technique actually decreases with increasing $c^{1/2}$. For similarly prepared samples, Green [3] finds K_c independent of $c^{1/2}$. However, when K_c decreases with increasing $c^{1/2}$, Green [3] suggests that this

